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THE RECUPERATOR ANALOGY FOR THE TRANSIENT PERFORMANCE OF THERMAL REGENERATORS

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Abstract—The cycle of operation of a thermal regenerator is discontinuous and consists of the passage of a hot fluid through the channels of the heat storing packing for a fixed length of time followed by the passage of a cold fluid, usually in the contra-flow direction through the same channels during another predetermined period. It is shown that a model of the equivalent thick-walled recuperator, which is continuous in operation, can be employed to calculate the transient response of a regenerator to step changes in operation. This continuous approach enables existing and possibly new transfer function models for recuperators to be applied to regenerators, and thence to the feedforward computer control of regenerator systems.

NOMENCLATURE

- A, heating surface area [m^2];
- C, specific heat of storing matrix [$J\,kg^{-1}$, deg K];
- $\bar{h}$, bulk heat transfer coefficient [$W\,m^{-2}$, deg K];
- K_p, coefficient of heat exchange [$W\,m^{-2}$, deg K];
- L, length of regenerator [m];
- M, mass of heat storing matrix [kg];
- m, mass of gas resident in heat exchanger [kg];
- P, length of operating period [s];
- Q_p, quantity of heat exchanged [$J\,m^{-2}$];
- S, specific heat of gas [$J\,kg^{-1}$, deg K];
- T, temperature of heat storing matrix [deg K];
- t, temperature of gas [deg K];
- W, mass flow rate of gas [$kg\,s^{-1}$];
- y, distance from regenerator entrance [m].

Greek symbols

- β, degree of unbalance $\Lambda'\Pi''/\Lambda''\Pi'$;
- γ_1, ratio of reduced periods Π''/Π';
- γ_2, ratio of reduced lengths Λ''/Λ';
- $\varepsilon g_1, \varepsilon g_2$, dimensionless measure of the transient response;
- η, ζ, dimensionless time;
- η_{REG}, thermal ratio;
- θ, time [s];
- Λ, reduced length $\bar{h}A/WS$ [dimensionless];
- ξ, dimensionless length;
- Π, reduced period $\bar{h}A(P-m/W)/MC$ [dimensionless].

Subscripts

- *in*, inlet;
- *x*, exit;
- *m*, mean;
- *H*, harmonic mean.

Superscripts

- $'$, refers to hot period;
- $''$, refers to cold period.

INTRODUCTION

THE REPRESENTATIVE differential equations of the dynamic behaviour of a thermal regenerator system were established by Nusselt [1] and Hausen [2]. These equations represent the transfer of heat to/from the fluid, usually a gas, passing through the channels of the regenerator packing or "chequerwork",

$$\bar{h}A(T-t) = WSL\frac{\partial t}{\partial y} + mS\frac{\partial t}{\partial \theta}, \tag{1}$$

and the storage of thermal energy in that chequerwork,

$$\bar{h}A(t-T) = MC\frac{\partial T}{\partial \theta}. \tag{2}$$

These equations are applied first to the *hot* period of operation during which heat is absorbed by the packing from the hot gas. At the *reversal*, a discontinuity in operation occurs when the hot gas is shut off and cold gas is passed through the same channels usually in the contra-flow direction for the duration of the *cold* period. The temperature behaviour of the regenerator in this cold period may be simulated by solving equations (1) and (2) again. The cold period is terminated by another reversal.

The discontinuity at the change-overs manifests itself in the mathematical model distinctly. Firstly, integration of the equations proceeds in the direction of gas flow and this is reversed at the end of each period for contra-flow operation. Secondly, the boundary conditions specify that the solid temperature distribution at the start of a period is equal to that at the end of the previous period. When the direction y is measured always in the direction of gas flow with origin at the gas entrance to the packing, these boundary conditions are written in the form of equations (3) and (4):

$$T''(y, 0) = T'(L-y, P') \quad (3)$$

$$T'(y, 0) = T''(L-y, P''). \quad (4)$$

In its simplest form, the model embodies the following assumptions:

(1) The heat transfer coefficients and thermal properties of fluids and solid packing do not vary with space and time in either the hot or the cold period of operation although the values in one period may be different from the opposite period;

(2) The gas flow rates are constant in each period;

(3) Longitudinal conduction of heat (in the direction of gas flow) in the solid packing is neglected;

(4) Internal resistance within the packing to heat transfer in a direction perpendicular to gas flow is embodied within a bulk heat transfer $\bar{h}$, in a manner proposed by Hausen [3] and discussed in detail by Willmott [4].

During each period of operation, the gas and solid temperatures in a regenerator vary with time. Although at cyclic equilibrium, which becomes manifest after many cycles of identical operation, the performance of the regenerator remains unchanged from one cycle to the next, it is common to describe this performance employing *time mean* exit gas temperatures. In particular, the thermal ratios η'_{REG} and η''_{REG} are used to measure the equilibrium effectiveness of the regenerator.

$$\eta'_{REG} = \frac{t'_{in} - t'_{x,m}}{t'_{in} - t''_{in}} \quad (5)$$

$$\eta''_{REG} = \frac{t''_{x,m} - t''_{in}}{t'_{in} - t''_{in}}. \quad (6)$$

Willmott and Burns [5], following the scheme of London *et al.* [6] employed these *time mean* exit gas temperatures to describe the response of the regenerator to step changes in operation. The hot side response εg_1 and the cold side response εg_2 are defined in equations (7) and (8) in terms of the time mean exit gas temperatures at cyclic equilibrium before the step change, $t'_{x,m}(0)$ and $t''_{x,m}(0)$, and when the new equilibrium is established following the step change in operation, $t'_{x,m}(\infty)$ and $t''_{x,m}(\infty)$. At the nth cycle following the step change, the time mean exit temperatures are $t'_{x,m}(n)$ and $t''_{x,m}(n)$ and the responses are:

$$\varepsilon g_1(n) = \frac{t'_{x,m}(n) - t'_{x,m}(0)}{t'_{x,m}(\infty) - t'_{x,m}(0)} \quad (7)$$

$$\varepsilon g_2(n) = \frac{t''_{x,m}(n) - t''_{x,m}(0)}{t''_{x,m}(\infty) - t''_{x,m}(0)}. \quad (8)$$

The use of these responses εg_1 and εg_2 in this form overcomes the difficulty of representing the superposition of changes in the thermal performance of a regenerator *between* one cycle and the next, and the continuously varying gas and solid temperatures *within* each and every cycle of operation. However the discontinuous nature of the thermal regenerator operation and its representative model makes it very difficult to apply conventional distributed system theory in order to develop transfer functions describing the transient performance of a regenerator following changes in operation. Such transfer functions should greatly facilitate the feedforward control of systems of regenerators in environments where the operating conditions or the thermal demands vary with time.

In contrast the hot and cold fluids pass simultaneously and continuously through the recuperative heat exchanger (or "recuperator"), separated by a partition wall through which heat is transferred from one fluid to another. Is it possible to predict regenerator dynamic behaviour using a continuous model of a recuperator?

If so, the transfer functions developed by Gilles [7], Burns *et al.* [8] and Burns [9] can be applied to regenerators as well as recuperative heat exchangers. Problems are known to exist in the regulation of large regenerator systems where operating conditions are unsteady; for example, Beets and Elshout [10] discuss the need to control by computer the Cowper stoves employed to preheat the air for blast furnaces.

Three factors determine the thermal inertia of the regenerator [5], namely: (i) the magnitude of the heat transfer coefficients; (ii) the ratio of the surface area of the packing of the regenerator available for heat transfer between gas and solid and the heat capacity flow rates of the gases passing through the regenerator; and (iii) the ratio of the same heating surface area and the thermal capacity of the regenerator packing. For a recuperator to have the same thermal inertia characteristics as a regenerator, one should expect the partition wall of the recuperator to have the same heat capacity as the packing of the regenerator (we therefore introduce the term "thick walled recuperator"), for the hot and cold gas flow rates to be the same for both devices and for the hot/cold period heat transfer coefficients for the regenerator to apply to the hot/cold gas streams for the equivalent recuperator. Most important, the area of the packing available for heat transfer between gas and solid in the regenerator should be replicated by the area of the partition wall exposed to the hot gas stream *and* by the area of the wall exposed to the cold gas stream. This implies that the total heating surface area for the regenerator is half that of the equivalent recuperator.

The mathematical model of the recuperator reflects the continuous operation of this type of heat exchanger. No discontinuities equivalent to the regenerator reversals are involved. Provided the dynamic characteristics of the regenerator and those of the equivalent recuperator are in some agreed sense the same, this continuous recuperator model can be used to predict the transient performance of a periodic flow regenerator. The computer simulation experiments reported later in this paper verify the equivalence of the conventional regenerator model and the recuperator model for regenerator transient behaviour calculations.

What is not immediately clear from these empirical considerations is how differences in the durations of the hot and cold periods, P' and P'' of regenerator operation can be embodied in the equivalent recuperator model. This will be discussed.

COMPARISONS BETWEEN THE GOVERNING EQUATIONS OF THE REGENERATOR AND THE RECUPERATOR

The equations in dimensionless form describing recuperator behaviour are [9]:

$$\frac{\partial T}{\partial \eta^*} = (t'-T)+\gamma_1(t''-T) \quad (9)$$

$$\frac{\partial t}{\partial \xi^*} = T-t' \quad (10)$$

$$\frac{\partial t''}{\partial \xi^*} = \gamma_2(t''-T), \quad (11)$$

where

$$\eta^* = \frac{\bar{h}'A}{MC}\theta \quad (12)$$

$$\xi^* = \frac{\bar{h}'A}{W'S'L}y \quad (13)$$

$$\gamma_1 = \bar{h}''/\bar{h}' \quad (14)$$

$$\gamma_2 = \Lambda''/\Lambda', \quad (15)$$

with

$$\Lambda' = \frac{\bar{h}'A}{W'S'} \quad (16)$$

$$\Lambda'' = \frac{\bar{h}''A}{W''S''}. \quad (17)$$

The corresponding equations for regenerators are yielded by applying the dimensionless parameters ξ and η to equations (1) and (2). These are

$$\frac{\partial t}{\partial \xi} = T-t \quad (18)$$

$$\frac{\partial T}{\partial \eta} = t-T. \quad (19)$$

Nusselt [1] and Rummel [11] suggested for cyclic equilibrium conditions that the limiting performance of the regenerator with zero period lengths is the same as that of the equivalent recuperator.

In the following analysis it will be shown that the equations governing the behaviour of the regenerator are equivalent to those of a corresponding recuperator, in the limit, as the period length tends to zero. This indicates the fundamental similarity between the two models. It is then assumed that regenerators operating with non-zero periods can be simulated approximately using this recuperator analogy. The acceptability of this approximation is verified by computer simulation.

It is important to recognize that "zero period" is a mathematically limiting condition and *not* a physical condition. It is assumed in this work that in taking this limit the cold/hot gas residing in the regenerator at the start of the hot/cold period is evacuated before the hot/cold gas starts to flow through the regenerator and furthermore, this evacuation of the cold/hot gas and the initial entry of new hot/cold gas at the beginning of the period does not influence the temperature of the regenerator packing. This must be distinguished from the physical conditions described by Heggs and Carpenter [12] and Willmott and Hinchcliffe [13] in which, as the period lengths are shortened, a body of gas is trapped in the packing thereby influencing the overall temperature behaviour of the regenerator.

Consider the 2-D equations (18) and (19) as they apply to the behaviour of a hot period.

$$\frac{\partial t'}{\partial \xi'} = T'-t' \quad (20)$$

$$\frac{\partial T'}{\partial \eta'} = t'-T'. \quad (21)$$

For a hot period of length Π' short enough, the local change in solid temperature $\Delta T'$ during the hot blow can be expressed approximately as

$$\Delta T' \simeq \left.\frac{\partial T}{\partial \eta'}\right|_{\eta'=0} \cdot \Pi', \quad (22)$$

where dimensionless time Π' is measured from the start of the hot period. Hence

$$\Delta T' = (t'-T')\Pi', \quad (23)$$

where t' and T' are the gas and solid temperatures at the start of the period. If t'' and T'' are the corresponding values at the start of the following cold period then it is possible to write

$$\Delta T'' = (t''-T'')\Pi''. \quad (24)$$

Now $T'' = T'+\Delta T'$; therefore if the product term $\Delta T'\Pi''$ can be neglected (being of order Δ^2, since both ΔT and Π'' are "small") then equation (24) can be rewritten as

$$\Delta T'' = (t''-T')\Pi''. \quad (25)$$

The total temperature change δT over a complete cycle of duration $\Pi'+\Pi''$ is given by $\delta T = \Delta T' + \Delta T''$. At cyclic equilibrium $\Delta T'+\Delta T''=0$. However in transience $\Delta T'+\Delta T''$ represents the change in solid temperature in time $\delta\Pi = \Pi'+\Pi''$. Let $r = \Pi'/(\Pi'+\Pi'')$ a constant. The average rate of solid temperature change as $\delta\Pi$ becomes very small is given by

$$\frac{\partial T}{\partial \eta'} = \lim_{\delta\Pi\to 0}\frac{\delta T}{\delta\Pi} = r(t'-T)+(1-r)(t''-T). \quad (26)$$

Finally a change of variable is made,

$$\zeta = r\eta', \quad (27)$$

in which case equation (26) takes the form

$$\frac{\partial T}{\partial \zeta} = (t'-T)+\frac{\Pi''}{\Pi'}(t''-T). \quad (28)$$

In the previous section the differential equation (9) describing the accumulation of thermal energy in the partition wall for the recuperator was presented:

$$\frac{\partial T}{\partial \eta^*} = (t'-T)+\gamma_1(t''-T), \quad (9)$$

where $\gamma_1 = \bar{h}''/\bar{h}'$.

A comparison between equations (28) and (9) reveals that in order to complete the mapping between the time scale used for the regenerator operation ζ and that used for the normal recuperator equivalent η^* a constant factor of $\Pi'/(\Pi'+\Pi'')$ needs to be employed. The use of ζ represents the concept in the model that the effect of the hot and cold gasses which pass alternatively in time $\Pi'+\Pi''$ over the regenerator packing, is achieved in time Π' in the equivalent recuperator model. The appearance of $\Pi''/\Pi' = \bar{h}''P''/\bar{h}'P'$ in equation (28) instead of $\bar{h}''/\bar{h}'$ in equation (9) represents a further normalisation which takes into account that the heat transfer coefficient $\bar{h}'$ is active in the regenerator for a period P', and $\bar{h}''$ for time P'' whereas in the recuperator both heat transfer coefficients are "active" on either side of the partition wall simultaneously.

The equations representing the behaviour of the gases in the regenerator model are:

$$\frac{\partial t'}{\partial \xi'} = T-t' \quad (20)$$

$$\frac{\partial t''}{\partial \xi''} = T-t'', \quad (29)$$

where

$$\xi' = \frac{\bar{h}'Ay'}{W'S'L}$$

$$\xi'' = \frac{\bar{h}''Ay''}{W''S''L},$$

with

$$y' = L-y''.$$

If equations (20) and (29) are both placed on the same dimensionless distance parameter ξ^*, where

$$\xi^* = \frac{\bar{h}'Ay'}{W'S'L},$$

then

$$\frac{\partial t''}{\partial \xi^*} = \frac{\partial t''}{\partial \xi''}\cdot\frac{\partial \xi''}{\partial y'}\cdot\frac{\partial y'}{\partial \xi} = \frac{\partial t''}{\partial \xi''}\left(-\frac{\Lambda''}{\Lambda'}\right). \quad (30)$$

But

$$\gamma_2 = \frac{\Lambda''}{\Lambda'}.$$

Therefore the system equations describing the limiting behaviour of a regenerator with zero period lengths are:

$$\frac{\partial t'}{\partial \xi^*} = T-t' \quad (31)$$

$$\frac{\partial t''}{\partial \xi^*} = \gamma_2(t''-T) \quad (32)$$

$$\frac{\partial T}{\partial \zeta} = (t'-T)+\frac{\Pi''}{\Pi'}(t''-T), \quad (33)$$

which are of identical form to those describing the recuperators transient behaviour (equations (9)–(11)).

The recuperator analogy is developed from considerations of the performance of a regenerator when the cycle time is very short. Under such circumstances, little variations of gas or solid temperature can take place within a cycle of regenerator operation. Consequently the gas and solid temperatures within the regenerator are equal to the corresponding gas and solid temperatures in the recuperator. In order to extend the application of the recuperator analogy to regenerators running with longer cycle times and where significant variations of gas and solid temperatures take place, it is necessary to assume:

(1) The local chronological average gas temperatures for the hot and cold periods in the regenerator are equivalent to hot and cold gas stream temperatures at the corresponding positions in the recuperator;

(2) The partition wall temperature, at any particular position in the recuperator, is equivalent to the time mean solid temperature at the same position in the regenerator, the average being taken over the whole cycle of operation.

In order to investigate the applicability of these assumptions it is necessary to develop a numerical solution for simulating the transient behaviour of the recuperator. The transient responses of the regenerator and its equivalent thick wall recuperator can then be compared over a wide range of parameters.

PREVIOUS USE OF RECUPERATOR ANALOGY

Although, as far as we can ascertain, a model of a recuperator equivalent to a regenerator in the form of the differential equations (31)–(33) has not been presented before, certainly a simple recuperator analogy of the regenerator was developed by Rummel [11] and Hausen [14, 3] for cyclic equilibrium calculations. Rummel and Hausen introduced a "coefficient of heat interchange" K_p between hot and cold gas in the recuperator, implicitly in the regenerator. Employing the bulk heat transfer coefficient $\bar{h}$ introduced by Hausen [3] and making the assumption that the temperature difference between gas and solid does not vary with time in the regenerator, and similarly that the temperature difference between hot gas and cold gas in the equivalent recuperator is time invariant one can specify that the quantity of heat Q_p exchanged in a complete period of regenerator operation per unit area should be at any position in the packing:

(1) Equal to the heat transferred per unit area from gas to solid in the hot period:

$$\bar{h}'(t'-T')P' = Q_p; \quad (34)$$

(2) Equal to the heat transferred per unit area from solid to gas in the cold period;

$$\bar{h}''(T''-t'')P'' = Q_p; \tag{35}$$

(3) Equal to heat transferred per unit area in time $P'+P''$ from hot to cold gas in the equivalent recuperator:

$$K_p(t'-t'')(P'+P'') = Q_p. \tag{36}$$

But at any position in the recuperator,

$$(t'-t'') = (t'-T')+(T''-t'')+(T'-T''). \tag{37}$$

Further, T' is equal to T'', the temperature of the partition wall in the equivalent recuperator at steady state. It follows that

$$\frac{Q_p}{K_p(P'+P'')} = \frac{Q_p}{\bar{h}'P'}+\frac{Q_p}{\bar{h}''P''},$$

or

$$\frac{1}{K_p} = \left\{\frac{1}{\bar{h}'P'}+\frac{1}{\bar{h}''P''}\right\}(P'+P''). \tag{38}$$

This scaling of the bulk heat transfer coefficients, namely $\bar{h}'P'/(P'+P'')$ and $\bar{h}''P''/(P'+P'')$ within the coefficient K_p corresponds to the appearance of $\bar{h}''P''/\bar{h}'P'$ in equation (33) instead of $\bar{h}''/\bar{h}'$ in equation (9). The development of equation (38) by Rummel and Hausen explicitly maps the transfer of heat within the hot and cold periods of regenerator operation onto a period $P'+P''$ of equivalent recuperator operation.

However this simple model of Rummel and Hausen excludes the possibility of studying theoretically the transient performance of the regenerator when it is not at cyclic equilibrium. The necessity does not arise therefore of developing a time scale of the form of ζ, which is required in the model set out here and which allows the chronological variations of the equivalent recuperator behaviour not at steady state to be related to the variations with time of the transient performance of the regenerator.

NUMERICAL SOLUTION OF RECUPERATOR ANALOGY EQUATIONS

Willmott [15] suggested a finite difference solution to the differential equations (18) and (14) for the regenerator. Two alternative developments of this method have been made for the recuperator equations (9), (10) and (11). Integration takes place over a grid, in the direction of hot gas flow ξ^* and in the time η^* direction. However, the boundary conditions are specified at $\xi^* = 0$ where the hot inlet gas temperature is t'_{in}, and at $\xi^* = \Lambda'$ where the cold gas inlet temperature is t''_{in}. As a consequence, the problem of obtaining numerical solutions to the differential equations is of the boundary value type.

Equations (9) and (10) can be reduced into the following forms employing the trapezoidal method:

$$t'_{r+1,s} = A'_1 t'_{r,s}+A'_2(T_{r+1,s}+T_{r,s}) \tag{39}$$

$$t''_{r+1,s} = A''_1 t''_{r+2,s}+A''_2(T_{r+1,s}+T_{r+2,s}), \tag{40}$$

where

$$A'_1 = \frac{2-\Delta\xi^*}{2+\Delta\xi^*};\quad A''_1 = \frac{2-\gamma_2\Delta\xi^*}{2+\gamma_2\Delta\xi^*}$$

$$A'_2 = \frac{\Delta\xi^*}{2+\Delta\xi^*};\quad A''_2 = \frac{\gamma_2\Delta\xi^*}{2+\gamma_2\Delta\xi^*}.$$

The suffix r,s refers to a position $r\Delta\xi^*$ from the hot gas entrance at time $s\Delta\xi^*$. The suffix $r+2$, appearing in equation (40), represents the contra flow of the cold gas with respect to the direction ξ^*.

In a similar way equation (9) can be manipulated using the trapezoidal rule:

$$T_{r,s+1} = B_1 T_{r,s}+B_2(t'_{r,s+1}+t'_{r,s}) +B_3(t''_{r,s+1}+t''_{r,s}), \tag{41}$$

where

$$B_1 = \frac{2-\Delta\eta^*(1+\gamma_1)}{2+\Delta\eta^*(1+\gamma_1)}$$

$$B_2 = \frac{\Delta\eta^*}{2+\Delta\eta^*(1+\gamma_1)}$$

$$B_3 = \gamma_1 B_2.$$

It is specified that $r = 0$ denotes the hot gas entrance and $r = m$, the cold gas entrance where $m\Delta\xi^* = \Lambda'$. The boundary conditions are therefore formulated:

(i) $t'_{0,s} = t'_{in}$ for all s
(ii) $t''_{m,s} = t''_{in}$
(iii) $T_{r,0}$ is defined arbitrarily for all r.

The boundary value problem can be solved through a matrix formulation, in which case the values of $t'_{r,s}$, $t''_{r,s}$ and $T_{r,s}$ ($r = 0,1,2,\ldots,m$) at the successive time intervals are obtained by solving a set of simultaneous linear equations.

Alternatively a shooting method can be employed in which integration in the ξ^* direction from the hot gas entrance is attempted. The inlet temperature of the hot gas stream is specified and provides one starting condition; in order to integrate "against the flow" of the cold gas stream, an estimate of the exit temperature of this cold gas is used as the other starting condition for the integration of the differential equations. The integration is repeated systematically at each time step and the cold gas exit temperature adjusted so that the correct inlet cold gas temperature is computed. Although the shooting method involves no matrix inversions, it requires up to 3 iterations per step in comparison with the explicit matrix method which is described below. A more detailed discussion of both methods is described by Burns [9].

Matrix formulation

Equations (39), (40) and (41) can be written in matrix form

$$U\mathbf{f}^{s+1} = V\mathbf{f}^s+\mathbf{C}. \tag{42}$$

The matrix U is

$$\begin{bmatrix} 1 & & & & & & & \\ -B_2 & 1 & -B_3 & & & & & \\ & -A_2'' & 1 & & 0 & -A_2'' & -A_1'' & \\ -A_1' & -A_2' & 0 & & 1 & -A_2' & & \\ & & & & -B_2 & 1 & -B_3 & \\ & & \cdots & & & & & \\ & & \cdots & & & & & \\ & & A_2'' & 1 & 0 & A_2'' & -A_1'' & \\ & -A_1' & A_2' & 0 & 1 & -A_2' & & \\ & & & & -B_2 & 1 & -B_3 & \\ & & & & & & 1 & \end{bmatrix}$$

and V is

$$\begin{bmatrix} 0 & 0 & & & & & \\ B_2 & B_1 & B_3 & & & & \\ & 0 & 0 & 0 & & & \\ & & 0 & 0 & 0 & & \\ & & & B_2 & B_1 & B_2 & \\ & & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & \cdot & \\ & & 0 & 0 & 0 & & \\ & & & 0 & 0 & 0 & \\ & & & & B_2 & B_1 & B_3 \\ & & & & & 0 & 0 \end{bmatrix}$$

Now $\mathbf{f}^s$ is the column vector of temperatures for time $s\Delta\eta^*$ from the start of the period under consideration where

$$\mathbf{f}^s = \{t'_{0,s}, T_{0,s}, t''_{0,s}, \cdots$$
$$\cdots t'_{r,s}, T_{r,s}, t''_{r,s} \cdots t'_{m,s}, T_{m,s}, t''_{m,s}\},$$

and $\mathbf{C}$ is the column vector containing the boundary conditions,

$$\mathbf{C} = \{t'_{in}, 0, 0, \ldots, 0, t''_{in}\}.$$

The integration procedure consists of initially calculating the gas temperatures on both sides of the partition wall at the start of the period under consideration employing equations (39) and (40) successively for $r = 0,1,2,\ldots,m-1$. For all subsequent time steps, the vector $\mathbf{f}^{s+1}$ is obtained by solving the matrix equation (42). At the start of the period, the wall temperature is specified arbitrarily or at the equilibrium conditions immediately prior to a step change in operation. Equation (42) can be formulated as

$$\mathbf{f}^{s+1} = U^{-1}V\mathbf{f}^s + U^{-1}\mathbf{C}.$$

So long as step changes in inlet gas temperatures are considered alone, the matrices $U^{-1}V$ and U^{-1} remain unaltered and need be computed just once in the calculation. However, with step changes in gas flow rate, these matrices must be re-computed and the speed of the method is greatly determined by the algorithm used to invert the band matrix U.

EFFECTIVENESS OF RECUPERATOR ANALOGY

Transient performance of regenerators

A number of computational experiments are now reported in which the transient performance predicted by the recuperator analogy is compared with that predicted by the regenerator model. In making the comparison, it is important to note that chronological *average* temperatures for hot/cold periods of regenerator model performance are compared with temperatures in the recuperator analogy at times equivalent to the *end* of the corresponding cycle of regenerator operation. This follows from the consideration that whereas the regenerator packing is subject sequentially to the effect of the hot and then the cold gas passing through the regenerator, the gases, both hot and cold, pass on either side of the recuperator partition wall simultaneously and that the effect of a whole set of cycles of regenerator operation will only become manifest in the recuperator analogy at a time equivalent to the end of such a set of cycles.

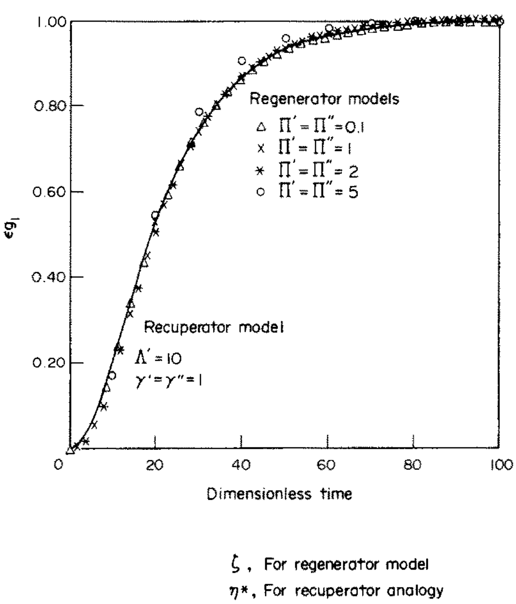

FIG. 1. Hot side response to a step change in hot side inlet gas temperature reduced length $\Lambda' = \Lambda'' = 10$.

The simplest case is the symmetric regenerator and Fig. 1 illustrates such a comparison where the hot side response (the response in hot gas exit gas temperature) to a step change in inlet temperature is considered for the case where the reduced length, $\Lambda' = \Lambda''$, is equal to 10. The continuous line illustrates the response predicted by recuperator analogy; the spot positions are responses predicted by the regenerator model employing reduced periods, $\Pi' = \Pi''$, equal to 0.1, 1.0, 2.0 and 5.0. Although the agreement between the models deteriorates as reduced period increases, as one might expect, the agreement between regenerator model and recuperator analogy is none-the-less excellent. Figure 2 provides a comparison for the cold side response to a step change in hot inlet temperature for the same

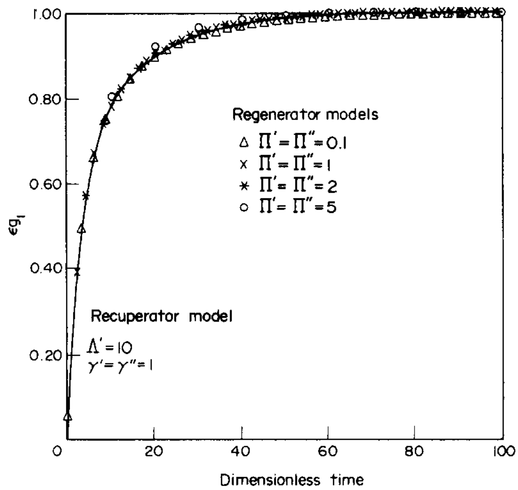

FIG. 2. Cold side response to a step change in hot side inlet gas temperature.

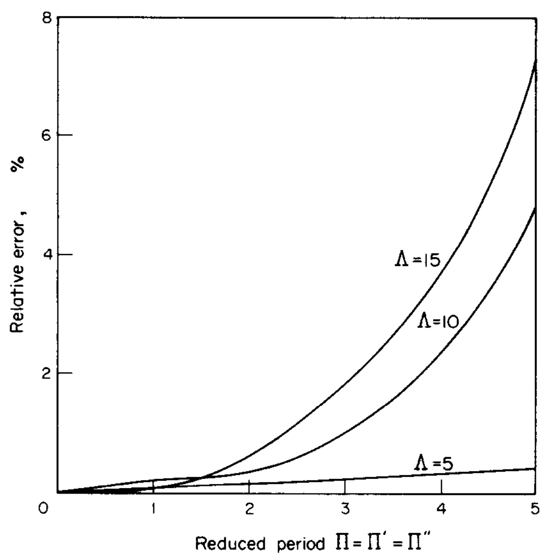

FIG. 3. Maximum relative error between the regenerative and recuperative models' symmetric configurations.

range of parameters. The agreement is much better in this case.

These two diagrams illustrate that for most of the response period until cyclic equilibrium or steady state performance is restored, it is difficult to distinguish between the response temperatures predicted by the regenerator or its recuperator analogy. We have therefore computed the *maximum* relative error over the duration of transient response to a hot inlet gas temperature step change, of the hot (and cold, but the largest error always occurs on the same side as that of the step change) exit temperature predicted by the recuperator analogy compared with that predicted by the regenerator model for a range of reduced periods $0 < \Pi \leqslant 5$ ($\Pi' = \Pi'' = \Pi$) and the reduced lengths 5, 10 and 15. (See Fig. 3.) It will be seen that this maximum relative error increases both with reduced length and reduced period.

Willmott and Burns [5] showed that the longer the reduced length, the more inert the regenerator system. It follows that as reduced length increases, the more protracted will be the response period and the greater the need to predict accurately the path of this response. None the less, a *maximum* relative error of less than 8% is quite good if it is borne in mind that for most of the response period, the relative error will be a good deal less. Further, these maximum errors occur near the beginning of the transient period when εg_1 and εg_2 are close to zero and when, as a consequence, small absolute errors become comparatively large relative errors.

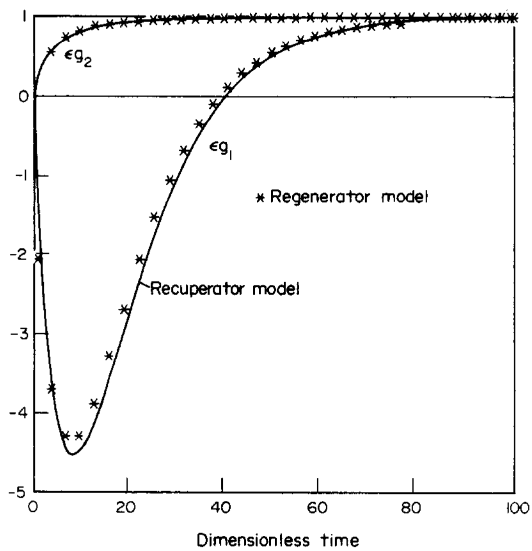

FIG. 4. Response to a simultaneous step change in hot inlet gas temperature and hot side gas flow rate.

Finally, the ability of the recuperator analogy to predict the effect of simultaneous step changes in hot inlet gas temperature and hot side flow rate is illustrated. Since it is assumed here that the heat transfer coefficient between gas and solid and gas flow rate are linearly proportional, the reduced length Λ' is not affected by the change in flow rate (see Burns [16]). For the regenerator model, the initial value of hot side reduced period is $\Pi'_{\mathrm{OLD}} = 3.0$ and the final value after the step change is $\Pi'_{\mathrm{NEW}} = 2.0$, representing the change in hot side flow rate. For the recuperator model, $\gamma_{1,\mathrm{OLD}} = \Pi''/\Pi'_{\mathrm{OLD}}$ is used before the step change and $\gamma_{1,\mathrm{NEW}} = \Pi''/\Pi'_{\mathrm{NEW}}$ is employed after the step change in equation (9). The resulting transient responses are illustrated in Fig. 4. Only the hot side response in the first three cycles of regenerator performance are predicted comparatively poorly by the recuperator model and it is clear that further work in this particular area is now required. None the less, the ability of the recuperator analogy to predict the overall response in this more complex situation is quite good.

CONCLUSIONS

It is becoming increasingly important to be able to regulate the operation of regenerators under conditions of time varying thermal load and input energy availability, and to do so with maximum thermal efficiency. Beets and Elshout [10] and Strausz [17] describe computer controlled Cowper stove systems based on conventional regenerator theory. In this paper we are looking towards computationally compact algorithms which can be implemented on microprocessor on-line control systems in which the likely performance of a regenerator following step changes in operation can be predicted and thus regulated. Burns *et al.* [8] and Gilles [7] show how conventional control theory can be applied to the recuperator analogy to yield transfer functions and thus simple predictive algorithms for regenerator performance. This paper establishes the foundations of this work by securing the relationship between the discontinuous model of the regenerator and the more useful continuous model of its recuperator equivalent.

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L'ANALOGIE AVEC LES RECUPERATEURS, DES REGENERATEURS THERMIQUES A FONCTIONNEMENT PERIODIQUE

Résumé—Le cycle opératoire d'un régénérateur thermique est discontinu et il est composé du passage d'un fluide chaud à travers les canaux de la matrice de stockage thermique pendant un certain laps de temps suivi par un temps de passage d'un fluide froid, généralement à contre-courant du premier passage et dans les mêmes canaux. On montre d'un modèle de récupérateur équivalent à paroi épaisse et à fonctionnement continu peut être employé pour calculer la réponse transitoire d'un régénérateur à des changements en échelon. Cette approche continue permet à des modèles existants, et à de nouvelles fonctions de transfert éventuelles, pour les récupérateurs, d'être appliqués aux régénérateurs et par suite d'être appliqués à leur commande en temps réel par des ordinateurs.

DIE REKUPERATOR-ANALOGIE ZUM INSTATIONÄREN VERHALTEN THERMISCHER REGENERATOREN

Zusammenfassung—Der Arbeitszyklus eines thermischen Regenerators ist diskontinuierlich und besteht aus der Strömung eines heißen Fluids durch die Kanäle der Wärmespeichermasse während eines festen Zeitintervalls, gefolgt von der Strömung eines kalten Fluids, normalerweise in Gegenrichtung, durch dieselben Kanäle während einer weiteren festgesetzten Periode. Es wird nun gezeigt, daß ein Modell des äquivalenten dickwandigen Rekuperators, der kontinuierlich arbeitet, verwendet werden kann, um die Übergangsfunktion eines Regenerators bei Änderung der Betriebsweise entsprechend einer Sprungfunktion zu berechnen. Der kontinuierliche Ansatz macht es möglich, daß vorhandene und möglicherweise neue Typen von Übertragungsfunktionen für Rekuperatoren auf Regeneratoren und damit auf die Prozeß regelung von Regeneratorsystemen angewendet werden können.

РЕКУПЕРАТОРНАЯ АНАЛОГИЯ РАСЧЁТА НЕСТАЦИОНАРНЫХ РЕЖИМОВ ТЕПЛОВЫХ РЕГЕНЕРАТОРОВ

Аннотация — Тепловой регенератор работает по прерывистому циклу, состоящему из протекания горячей жидкости через каналы теплоаккумулирующей насадки за фиксированный промежуток времени с последующим протеканием холодной жидкости, обычно в противоположном направлении, через те же каналы в течение другого заданного промежутка времени. Показано, что модель эквивалентного толстостенного рекуператора, работающего в непрерывном режиме, может быть использована для расчёта нестационарных режимов регенератора при ступенчатом изменении условий его работы. Этот подход позволяет применить для расчёта регенераторов имеющиеся, а также новые функциональные модели переноса в рекуператорах. Результаты расчётов могут быть использованы затем для составления программ ЭВМ для контроля за работой регенераторных систем.